Italo Bongioanni

Fundamental Trigonometric Identities

Reciprocal Identities

$\sin x = \frac{1}{\csc x}$ $\quad \cos x = \frac{1}{\sec x}$ $\quad \tan x = \frac{1}{\cot x}$

$\csc x = \frac{1}{\sin x}$ $\quad \sec x = \frac{1}{\cos x}$ $\quad \cot x = \frac{1}{\tan x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$ $\quad 1 + \tan^2 x = \sec^2 x$ $\quad 1 + \cot^2 x = \csc^2 x$

Quotient Identities

$\tan x = \frac{\sin x}{\cos x}$ $\quad \cot x = \frac{\cos x}{\sin x}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{4\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

Co-Function Identities

$\sin(\frac{\pi}{2} - x) = \cos x$ $\quad \cos(\frac{\pi}{2} - x) = \sin x$ $\quad \tan(\frac{\pi}{2} - x) = \cot x$

$\csc(\frac{\pi}{2} - x) = \sec x$ $\quad \sec(\frac{\pi}{2} - x) = \csc x$ $\quad \cot(\frac{\pi}{2} - x) = \tan x$

Even-Odd Identities

$\sin(-x) = -\sin x$ $\quad \cos(-x) = \cos x$ $\quad \tan(-x) = -\tan x$

$\csc(-x) = -\csc x$ $\quad \sec(-x) = \sec x$ $\quad \cot(-x) = -\cot x$

Sum and Difference Identities

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$ $\quad \sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$ $\quad \sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$ $\quad \tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

Double Angle Identities

$\sin 2x = 2\sin x\cos x$ $\quad \cos 2x = \cos^2 x - \sin^2 x$

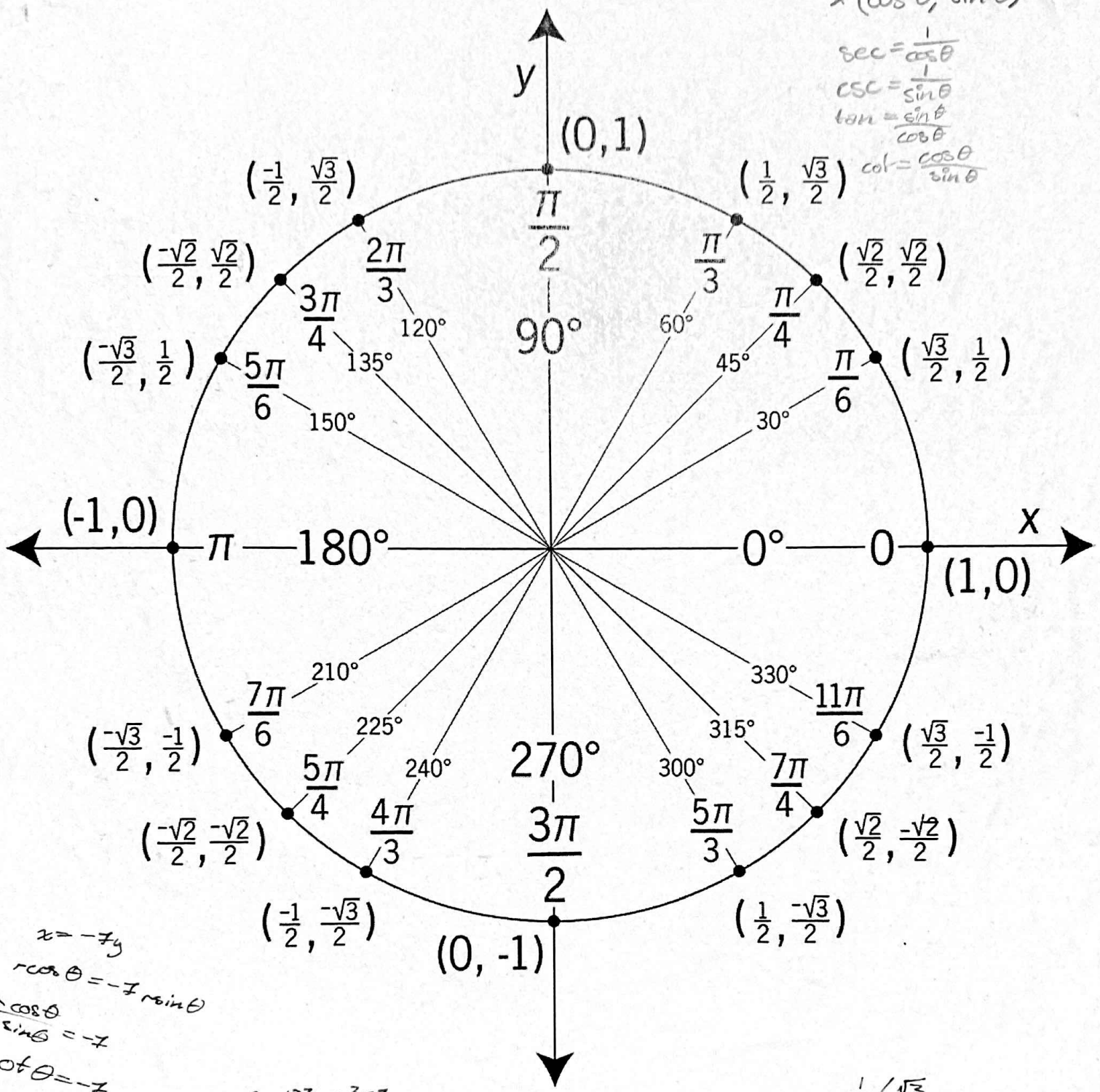
$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$ $\quad \cos 2x = 2\cos^2 x - 1$

$\cos 2x = 1 - 2\sin^2 x$

Power Reducing Identities

$\sin^2 x = \frac{1 - \cos 2x}{2}$ $\quad \cos^2 x = \frac{1 + \cos 2x}{2}$

Unit Circle



$\star (\cos\theta, \sin\theta)$

$\sec = \frac{1}{\cos\theta}$

$\csc = \frac{1}{\sin\theta}$

$\tan = \frac{\sin\theta}{\cos\theta}$

$\cot = \frac{\cos\theta}{\sin\theta}$

$x = -7y$

$r\cos\theta = -7 r\sin\theta$

$\frac{r\cos\theta}{r\sin\theta} = -7$

$\cot\theta = -7$

$r = 2\sec 2\theta \cos\theta \rightarrow (x-1)^2 - y^2 = 1$

$r = 2\frac{1}{\cos^2\theta - \sin^2\theta}\cos\theta \quad (r\cos\theta - 1)^2 - (r\sin\theta)^2 = 1$

$r^2\cos^2\theta - 2r\cos\theta + 1 - r^2\sin^2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) - 2r\cos\theta = 0$

$r^2(\cos^2\theta - \sin^2\theta) = 2r\cos\theta$

$r(\cos^2\theta - \sin^2\theta) = 2\cos\theta$

$r = 2\frac{\cos\theta}{\cos^2\theta - \sin^2\theta} \rightarrow r = 2\frac{1}{\cos^2\theta - \sin^2\theta} \cdot \cos\theta$

$r = 2\sec 2\theta \cos\theta$

$\frac{1}{2} / \frac{\sqrt{3}}{2}$

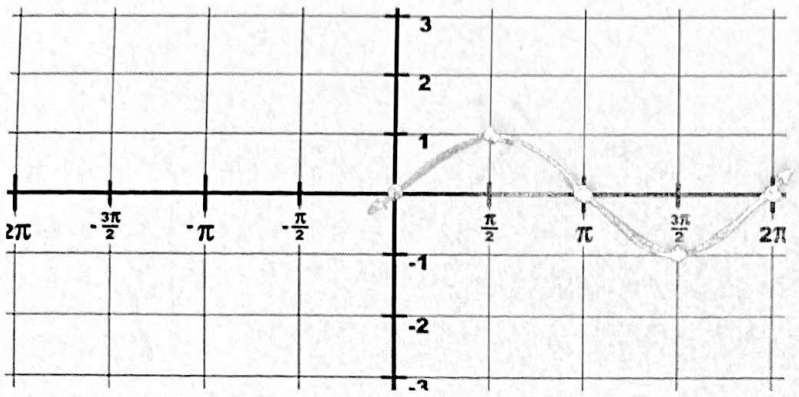
$\rightarrow \frac{1}{2} \cdot \frac{2}{\sqrt{3}}$

$\rightarrow \frac{\sqrt{3}}{3}$

Italo Bongioanni B1

$f(x) = \sin x$

x	$f(x)$
0	0
$\frac{\pi}{2}$	1
π	0
$\frac{3\pi}{2}$	-1
2π	0



Start/End: $[0, 2\pi]$

Period: 2π

Domain: $(-\infty, \infty)$

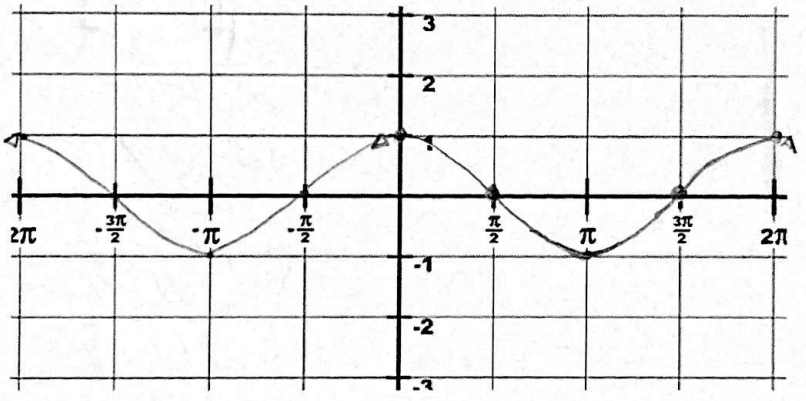
Range: $[-1, 1]$

x-int: $0 + \pi n, n \in \mathbb{Z}$

y-int: $(0, 0)$

$f(x) = \cos x$

x	$f(x)$
0	1
$\frac{\pi}{2}$	0
π	-1
$\frac{3\pi}{2}$	0
2π	1



Start/End: $[0, 2\pi]$

Period: 2π

Domain: $(-\infty, \infty)$

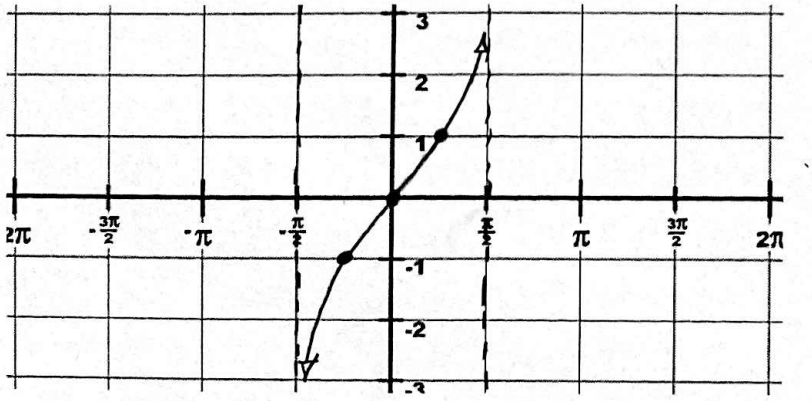
Range: $[-1, 1]$

x-int: $\frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

y-int: $(0, 1)$

$f(x) = \tan x$

x	$f(x)$
$-\pi/2$	VA
$-\pi/4$	-1
0	0
$\pi/4$	1
$\pi/2$	VA



Start/End: $(-\frac{\pi}{2}, \frac{\pi}{2})$

Period: π

Domain: $x \neq \frac{\pi}{2} + n\pi, n \in \mathbb{Z}$

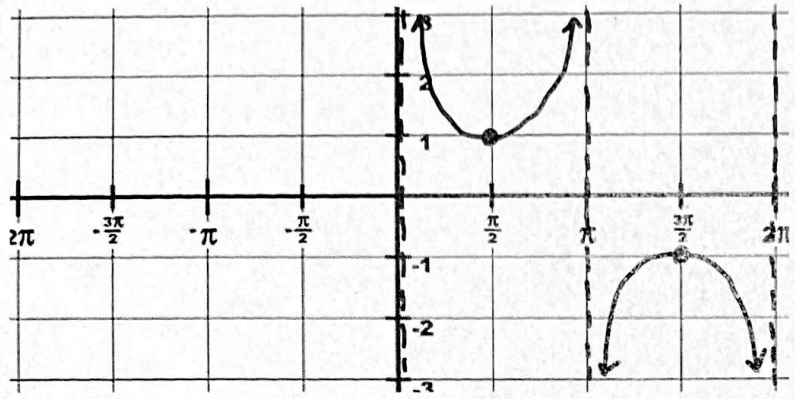
Range: $(-\infty, \infty)$

x-int: $n\pi, n \in \mathbb{Z}$

y-int: 0

$f(x) = \csc x$

x	y
0	VA
$\frac{\pi}{2}$	1
π	VA
$\frac{3\pi}{2}$	-1
2π	VA



Start/End: $(0, 2\pi)$

Period: 2π

Domain: $x \neq \pi n, n \in \mathbb{Z}$

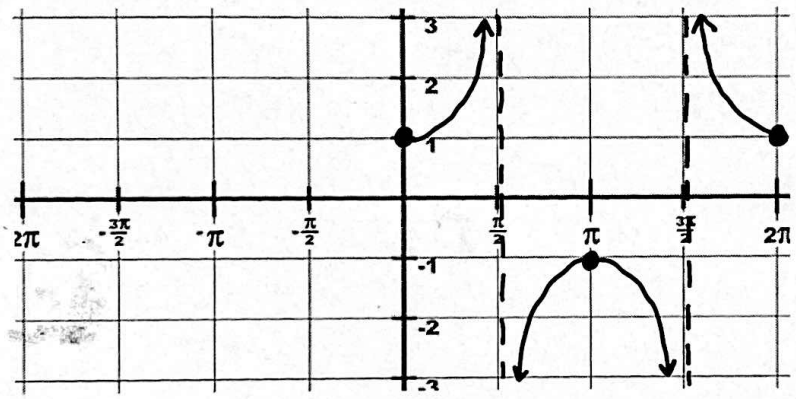
Range: $(-\infty, -1] \cup [1, \infty)$

x-int: none

y-int: none

$f(x) = \sec x$

x	y
0	1
$\frac{\pi}{2}$	VA
π	-1
$\frac{3\pi}{2}$	VA
2π	1



Start/End: $[0, 2\pi]$

Period: 2π

Domain: $x \neq \frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

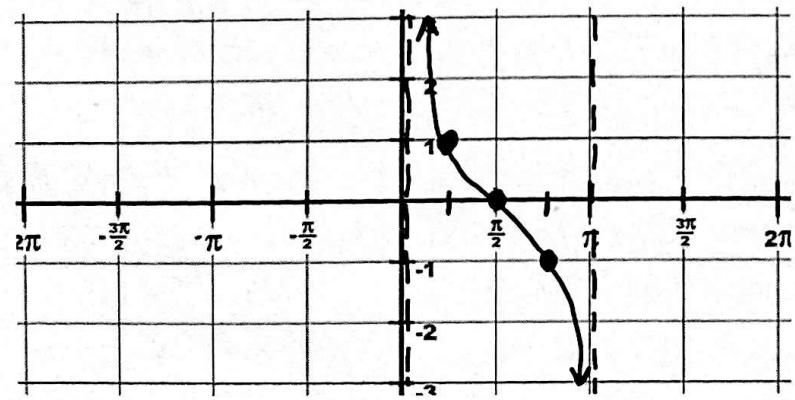
Range: $(-\infty, -1] \cup [1, \infty)$

x-int: none

y-int: $(0, 1)$

$f(x) = \cot x$

x	y
0	VA
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3\pi}{4}$	-1
π	VA



Start/End: $(0, \pi)$

Period: π

Domain: $x \neq n\pi, n \in \mathbb{Z}$

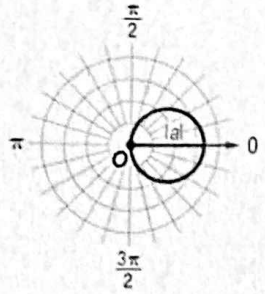
Range: $(-\infty, \infty)$

x-int: $\frac{\pi}{2} + \pi n, n \in \mathbb{Z}$

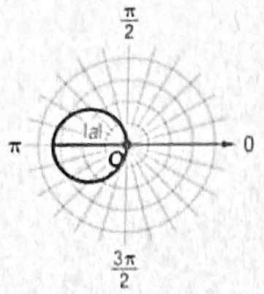
y-int: none

Circles

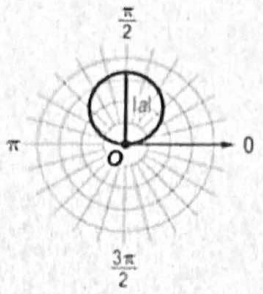
$r = a\cos\theta$ or $r = a\sin\theta$



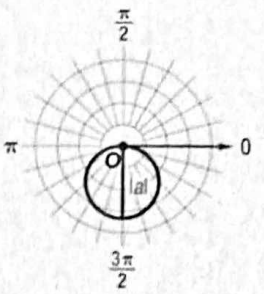
$a > 0$
$r = a\cos\theta$



$a < 0$
$r = a\cos\theta$



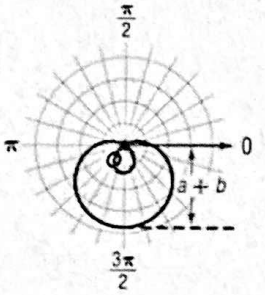
$a > 0$
$r = a\sin\theta$



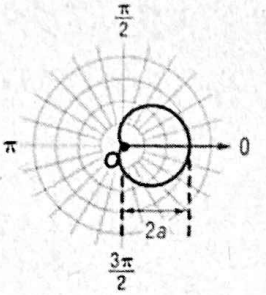
$a < 0$
$r = a\sin\theta$

Limaçons

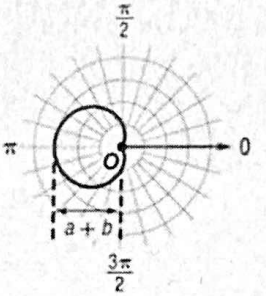
$r = a \pm b\cos\theta$ or $r = a \pm b\sin\theta$, where a and b are both positive



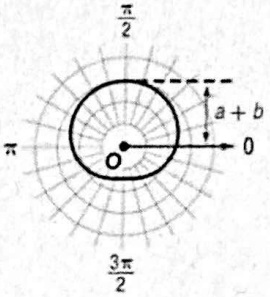
Limaçon with inner loop
$a < b$
$r = a - b\sin\theta$



Cardioid
$a = b$
$r = a + b\cos\theta$



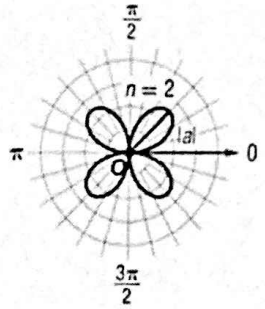
Dimpled limaçon
$b < a < 2b$
$r = a - b\cos\theta$



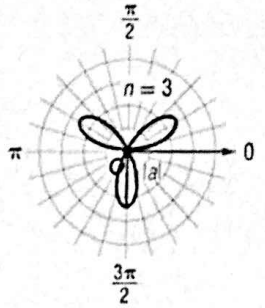
Convex limaçon
$a \geq 2b$
$r = a + b\sin\theta$

Roses

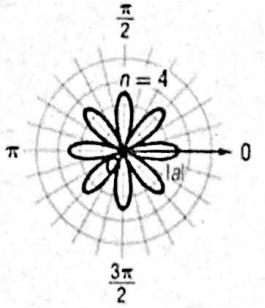
$r = a\cos n\theta$ or $r = a\sin n\theta$, where $n \geq 2$ is an integer
The rose has n petals if n is odd and $2n$ petals if n is even.



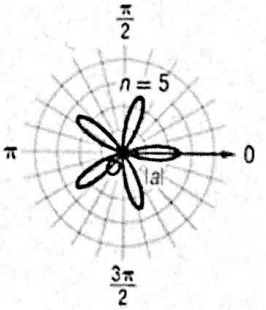
$r = a\sin n\theta$



$r = a\sin n\theta$



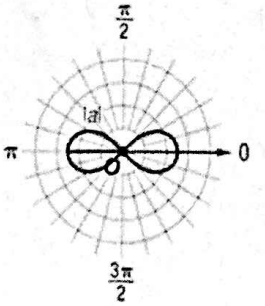
$r = a\cos n\theta$



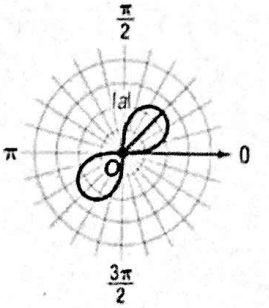
$r = a\cos n\theta$

Lemniscates (LEM-nis-keyts)

$r^2 = a^2\cos 2\theta$ or $r^2 = a^2\sin 2\theta$



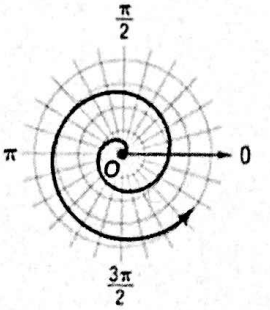
$r^2 = a^2\cos 2\theta$



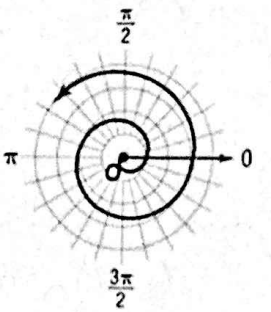
$r^2 = a^2\sin 2\theta$

Spirals of Archimedes (ahr-kuh-MEE-deez)

$r = a\theta + b$



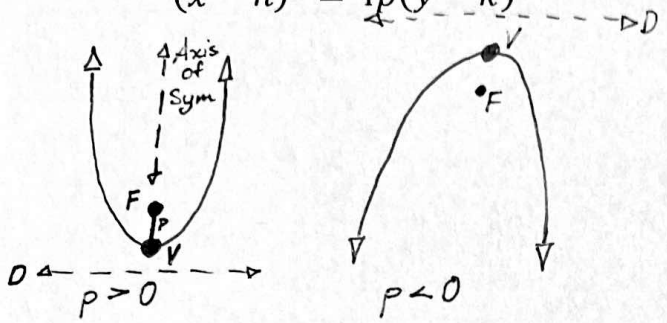
$\theta > 0$, $r = a\theta + b$



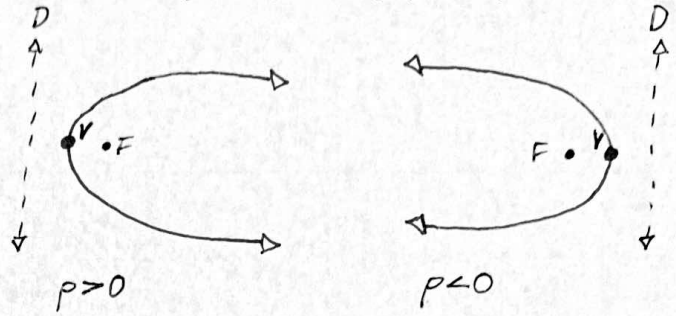
$\theta < 0$, $r = a\theta + b$

Parabolas

$(x-h)^2 = 4p(y-k)$



$(y-k)^2 = 4p(x-h)$



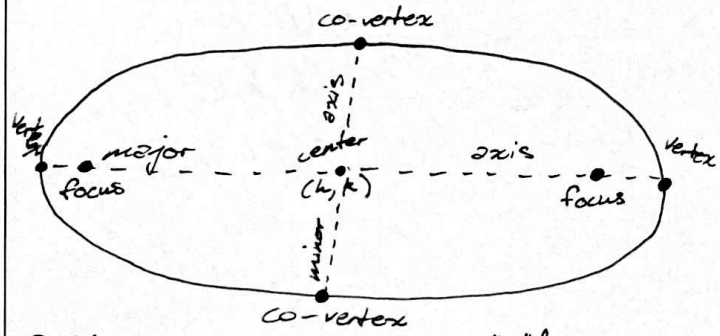
All points are equidistant between focus & directrix

$p > 0$ ↑ $p \to$ $p = 0$

Ellipses

"a" is always bigger

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

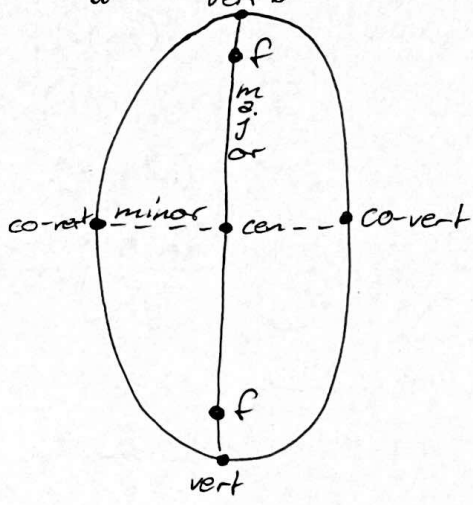


C → V = a C → CV = b C → F = "C" length

V → V = 2a CV → CV = 2b

$$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$$

vert

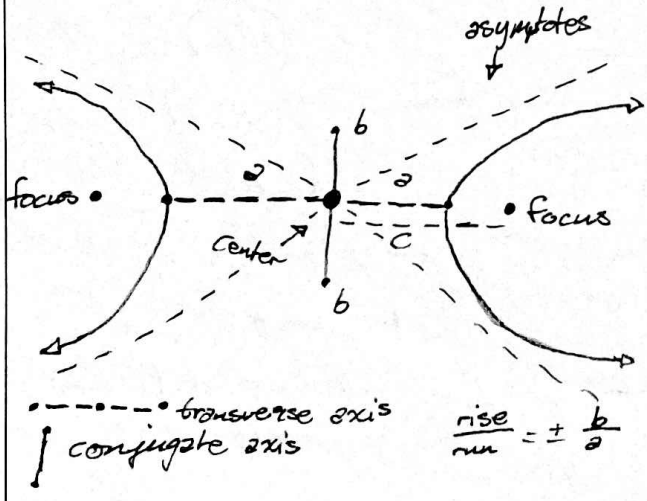


Circle $(x-h)^2 + (y-k)^2 = r^2$

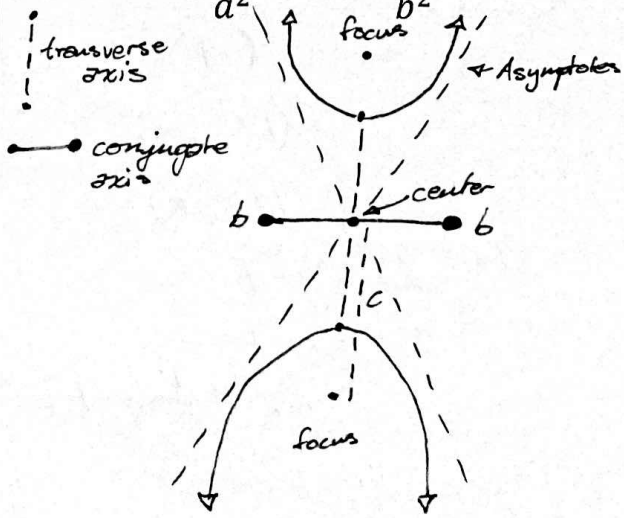
Hyperbolas

"a" is always first

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$



$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$



Italo Bongiovanni B1

Parabolas

$(x-h)^2 = 4p(y-k)$	
Vertex	(h, k)
Focus	$(h, k+p)$
Axis of Sym.	$x=h$
Directrix	$y=k-p$

$(y-k)^2 = 4p(x-h)$	
Vertex	(h, k)
Focus	$(h+p, k)$
Axis of Sym.	$y=k$
Directrix	$x=h-p$

Ellipses

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	
Center	(h, k)
Foci	$(h \pm c, k)$
Vertices	$(h \pm a, k)$
Co-Vertices	$(h, k \pm b)$
Major Axis $2a$	$y=k$
Minor Axis $2b$	$x=h$
a, b, c relationship	$c^2 = b^2 - a^2$

$\frac{(y-k)^2}{a^2} + \frac{(x-h)^2}{b^2} = 1$	
Center	(h, k)
Foci	$(h, k \pm c)$
Vertices	$(h, k \pm a)$
Co-Vertices	$(h \pm b, k)$
Major Axis $2a$	$x=h$
Minor Axis $2b$	$y=k$
a, b, c relationship	$c^2 = b^2 - a^2$

Circle $(x-h)^2 + (y-k)^2 = r^2$

Hyperbolas

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	
Center	(h, k)
Vertices	$(h \pm a, k)$
Foci	$(h \pm c, k)$
Transverse Axis $2a$	$y=k$
Conjugate Axis $2b$	$x=h$
Asymptotes	$y-k = \pm \frac{b}{a}(x-h)$
a, b, c relationship	$a^2 + b^2 = c^2$

$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$	
Center	(h, k)
Vertices	$(h, k \pm a)$
Foci	$(h, k \pm c)$
Transverse Axis $2a$	$x=h$
Conjugate Axis $2b$	$y=k$
Asymptotes	$y-k = \pm \frac{a}{b}(x-h)$
a, b, c relationship	$a^2 + b^2 = c^2$